Schaum’s Outlines, Essential computer mathematics by Seymour Lipschutz, solutions to problems

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Abstract

This document contains the solutions to problems posed in the book 'Theory and Problems of Essential Computer Mathematics' by Seymour Lipschutz (Schaum’s outline series). ISBN 0-07-037990-4. Solutions by Élie De Brauwer (elie@de-brauwer.be), this document originated from www.de-brauwer.be where the most recent version of this document can be downloaded. For any comments, errors, or anything else you may always contact me. I am not responsible for any errors or mistakes that can and probably will arise in this document. This document was created using L\TeX\ and emacs running under Debian Linux on a Sun Blade 100 system.
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1 Binary Number System

1.1 Decimal System

(1.29) Evaluate:

(a) $10^4$

$10^4 = 10 \times 10 \times 10 \times 10 = 10000$

(b) $10^{-3}$

$10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0.001$

(c) $10^{-1}$

$10^{-1} = \frac{1}{10} = 0.1$

(d) $2^6$

$2^6 = 2^3 \cdot 2^3 = 64$

(e) $2^{-3}$

$2^{-3} = \frac{1}{8} = 0.125$

(f) $2^0$

$2^0 = 1$

(g) $2^3$

$2^3 = 8$

(1.30) Evaluate:

(a) $8^3$

$8^3 = 8 \cdot 8 \cdot 8 = 512$

(b) $8^0$

$8^0 = 1$

(c) $8^{-2}$

$8^{-2} = \frac{1}{64}$

(d) $16^2$

$16^2 = 256$

(e) $16^{-1}$

$16^{-1} = \frac{1}{16}$

(f) $16^{-3}$

$16^{-3} = \frac{1}{4096}$

(1.31) Give the place value of each underlined digit

(a) 44333

$= 4 \cdot 10^3$

Place value equals 1000

(b) 22555.66

$= 5 \cdot 10^2$

Place value equals 100
1.1 Decimal System

(c) \(444,555\)
\[
= 5 \cdot 10^{-2}
\]
Place value equals 0,01

(d) \(22,33455\)
\[
= 4 \cdot 10^{-4}
\]
Place value equals 0,0001

(1.32) Write in expanded notation

(a) \(13579\)
\[
13579 = 1 \cdot 10^4 + 3 \cdot 10^3 + 5 \cdot 10^2 + 7 \cdot 10^1 + 9 \cdot 10^0
\]
(b) \(321,789\)
\[
321,789 = 3 \cdot 10^2 + 2 \cdot 10^1 + 1 \cdot 10^0 + 7 \cdot 10^{-1} + 8 \cdot 10^{-2} + 9 \cdot 10^{-3}
\]

(1.33) Find each sum

(a) \(835, 24 + 70, 456\)
\[
835, 24 + 70, 456 = 905, 696
\]
(b) \(55, 5 + 6, 66 + 0, 777\)
\[
55, 5 + 6, 66 + 0, 777 = 62, 937
\]

(1.34) Find each difference

(a) \(456, 7 − 35, 79\)
\[
456, 7 − 35, 79 = 420, 91
\]
(b) \(12 − 4, 888\)
\[
12 − 4, 888 = 7, 112
\]

(1.35) Find each product

(a) \(38, 24 \cdot 3, 7\)
\[
38, 24 \cdot 3, 7 = 141, 488
\]
(b) \(0, 0345 \cdot 1, 02\)
\[
0, 0345 \cdot 1, 02 = 0, 03519
\]

(1.36) Evaluate to two decimal places

(a) \(36 ÷ 11\)
\[
36 ÷ 11 = 3, 27
\]
(b) \(83, 472 ÷ 2, 4\)
\[
83, 472 ÷ 2, 4 = 34, 78
\]

elie@de-brauwer.be
1.2 Binary System

(1.37) Give the place value of each underlined bit

(a) \(111000\)
\[= 1 \cdot 2^3\]
Place value equals \(2^3\)

(b) \(1001100\)
\[= 0 \cdot 2^5\]
Place value equals \(2^5\)

(c) \(111,000111\)
\[= 0 \cdot 2^{-2}\]
Place value equals \(2^{-2}\)

(d) \(11,00110011\)
\[= 0 \cdot 2^{-5}\]
Place value equals \(2^{-5}\)

(1.38) Rewrite in expanded notation

(a) \(11001100\)
\[11001100 = 1 \cdot 10^7 + 1 \cdot 10^6 + 0 \cdot 10^5 + 0 \cdot 10^4 + 1 \cdot 10^3 + 1 \cdot 10^2 + 0 \cdot 10^1 + 0 \cdot 10^0\]

(b) \(111,000111\)
\[111,000111 = 1 \cdot 10^2 + 1 \cdot 10^1 + 1 \cdot 10^0 + 0 \cdot 10^{-1} + 0 \cdot 10^{-2} + 0 \cdot 10^{-3} + 1 \cdot 10^{-4} + 1 \cdot 10^{-5} + 1 \cdot 10^{-6}\]

(1.39) Convert each binary number to its decimal equivalent

(a) \(110110\)
\[110110 = 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^2 + 1 \cdot 2\]
\[110110 = 32 + 16 + 4 + 2\]
\[110110 = 54\]

(b) \(111000111\)
\[111000111 = 1 \cdot 2^8 + 1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^4 + 1 \cdot 2^1 + 1 \cdot 2^0\]
\[111000111 = 256 + 128 + 64 + 4 + 2 + 1\]
\[111000111 = 455\]

(1.40) Convert to its decimal equivalent

(a) \(110,11\)
\[110,11 = 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^{-1} + 1 \cdot 2^{-2}\]
\[110,11 = 4 + 2 + 0.5 + 0.25\]
\[110,11 = 5.75\]

(b) \(1010,10101\)
\[1010,10101 = 1 \cdot 2^4 + 1 \cdot 2^1 + 1 \cdot 2^{-1} + 1 \cdot 2^{-3} + 1 \cdot 2^{-5}\]
1.2 Binary System

1010, 10101 = 8 + 2 + 0, 5 + 0, 125 + 0, 03125
1010, 10101 = 10, 65625

(1.41) Convert each decimal number to its binary equivalent

(a) 285
285 = 256 + 16 + 8 + 4 + 1
285 = 100011101

(b) 473
473 = 256 + 128 + 64 + 16 + 8 + 1
473 = 111011001

(c) 694
694 = 512 + 128 + 32 + 16 + 4 + 2
694 = 1010110110

(1.42) Convert to its binary equivalent

(a) 0, 390625
0, 390625 = 0, 011001

(b) 24, 625
24, 625 = 11000, 101

(c) 0, 8
0, 8 = 0, 110011001100...

(d) 0, 3
0, 3 = 0, 0100110011...

(1.43) Show that the decimal equivalent of a terminating binary fraction also terminates (in a 5)

First take a look at the following piece of C++ source code. It merely prints $\forall x \in \{0, 1, 2, 3, 4, 5, 6, 7\}; \frac{1}{2^x}$.

```
#include <iostream>

int main(){
    double result=1;
    for(int i=0;i<8;i++){
        result/=2.0;
        cout << "2^" << (-1* i) << " = " << result << endl;
    }
}
```

And it’s output.
1.3 Binary Arithmetic

\[2^0 = 0.5\]
\[2^{-1} = 0.25\]
\[2^{-2} = 0.125\]
\[2^{-3} = 0.0625\]
\[2^{-4} = 0.03125\]
\[2^{-5} = 0.015625\]
\[2^{-6} = 0.0078125\]
\[2^{-7} = 0.00390625\]

This output is remarkable, you can see the last number is always 5, the number before that is always 2, the number before alternates between 1 and 6, the number before that alternates between 0, 3, 5, 8.
So, for each \(n\) gives \(\frac{1}{2^n}\) and ending decimal fraction, when you take the sum of an amount of \(x\) ending decimal fractions, the sum will also end.

1.3 Binary Arithmetic

(1.44) Find the binary sums.

(a) \(1101 + 111\)

\[
\begin{array}{c}
  1101 \\
  + 111 \\
  \hline
  10100
\end{array}
\]

(b) \(110011 + 11101\)

\[
\begin{array}{c}
  110011 \\
  + 11101 \\
  \hline
  1010000
\end{array}
\]

(c) \(11100111 + 11000011\)

\[
\begin{array}{c}
  11100111 \\
  + 11000011 \\
  \hline
  110101010
\end{array}
\]

(d) \(110.1101 + 1011.101\)

\[
\begin{array}{c}
  110.1101 \\
  + 1011.101 \\
  \hline
  10010.0111
\end{array}
\]
1.3 Binary Arithmetic

(1.45) Find the binary sums.

(a) \(11001 + 11100 + 1011 + 110011\)

\[
\begin{array}{c}
11001 \\
11100 \\
1011 \\
110011 \\
\hline
1110011
\end{array}
\]

(b) \(11.101 + 110.01 + 111.101 + 1101.1\)

\[
\begin{array}{c}
11.101 \\
110.01 \\
111.101 \\
1101.1 \\
\hline
11111.000
\end{array}
\]

(1.46) Find the binary products.

(a) \(11100111 \times 11\)

\[
\begin{array}{c}
11100111 \\
x 11 \\
\hline
11100111 \\
11100111 \\
\hline
1010110101
\end{array}
\]

(b) \(111011 \times 1011\)

\[
\begin{array}{c}
111011 \\
x 1011 \\
\hline
111011 \\
111011 \\
000000 \\
111011 \\
\hline
1010001001
\end{array}
\]
1.3 Binary Arithmetic

(c) $11.101 \times 11.01$

\[
\begin{align*}
11.101 \\
x11.01 \\
\hline
0.11101 \\
11.101 \\
111.01 \\
+\hline
1011.11001
\end{align*}
\]

(1.46) Find the binary differences.

(a) $1100011 - 110111$

\[
\begin{align*}
1100011 \\
110111 \\
\hline
101100
\end{align*}
\]

(b) $10101010 - 110011$

\[
\begin{align*}
10101010 \\
110011 \\
\hline
1110111
\end{align*}
\]

(c) $110.001 - 11.111$

\[
\begin{align*}
110.001 \\
11.111 \\
\hline
10.010
\end{align*}
\]

(1.48) Find the binary quotients.

(a) $1011011 \div 111$

\[
\begin{align*}
1011011 & \div 111 \\
111 & \underline{-111} \\
111 & \underline{111} \\
\hline
111 & \underline{111} \\
111 & \underline{111} \\
\hline
\text{R} & 0
\end{align*}
\]
1.4 Complements

(b) $100.0001 \div 10.1$

\[
\begin{array}{c|c}
1000.001 & 101 \\
- \hline
101 & 1.101 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c}
11.0 & 10.1 \\
- \hline
0.101 & 0.101 \\
\hline
0 & 0
\end{array}
\]

(c) $1011 \div 11$

\[
\begin{array}{c|c}
1011.000000 & 11 \\
- \hline
11 & 11.10101... \\
\hline
\end{array}
\]

\[
\begin{array}{c|c}
101 & 11 \\
\hline
10.0 & 1.1 \\
\hline
0.100 & 0.011 \\
\hline
.00100 & \\
\hline
\end{array}
\]

1.4 Complements

(1.49) Determine the nines and tens complements of the decimal numbers.

(a) $3201$

Nines complement : 6798
Tens complement : 6799

(b) $453,800$

Nines complement : 546,199
Tens complement : 546,200

(c) $78,923,019$

Nines complement : 21,076,980
Tens complement : 21,076,981
1.4 Complements

(d) 3 334 455 566
   Nines complement : 6 665 544 433
   Tens complement: 6 665 544 434

(1.50) If the numbers of Problem 1.49 are input to an 8-place decimal calculator, find their nines and tens complements in the machine.

(a) 3201
   Nines complement: 99 996 798
   Tens complement: 99 996 799

(b) 453 800
   Nines complement : 99 546 199
   Tens complement : 99 546 200

(c) 78 923 019
   Nines complement : 21 076 980
   Tens complement : 21 076 981

(d) 3 334 455 566
   Nines complement : 65 544 433
   Tens complement: 65 544 434

(1.51) Find the ones and twos complements of the binary numbers.

(a) 110011
   Ones complement: 001100
   Twos complement: 001101

(b) 10001000
   Ones complement: 01110111
   Twos complement: 01111000

(c) 10111011101
   Ones complement: 01000100010
   Twos complement: 01000100011

(d) 111000000111
   Ones complement: 000111111000
   Twos complement: 000111111001

(1.52) Find the following binary differences using complements.
   Note: I will use the ones complement method in all these examples.

(a) 1101 - 110
   1101
   1001
   +———
   10110
   +1 (carry)
1.4 Complements

\[\begin{align*}
\text{(b) } 11100111 - 11001100 & \\
11100111 \\
00110011 & + \ldots \\
100011010 & + 1 \text{ (carry)} \\
& \ldots \\
& 00011011
\end{align*}\]

\[\begin{align*}
\text{(c) } 11000010 - 10111001 & \\
11000010 \\
01000110 & + \ldots \\
100001000 & + 1 \text{ (carry)} \\
& \ldots \\
& 00001001
\end{align*}\]

\[\begin{align*}
\text{(d) } 10101 - 11011 & \\
10101 \\
00100 & + \ldots \\
11001 & \text{No carry thus -00110 (the negative ones complement is the solution in this case.)}
\end{align*}\]
2 Computer Codes

2.1 Number systems

(2.49) Write in expanded notation.

(a) \[2043_6 = 2 \times 6^3 + 0 \times 6^2 + 4 \times 6^1 + 3 \times 6^0\]

(b) \[435.621_7 = 4 \times 7^2 + 3 \times 7^1 + 5 \times 7^0 + 6 \times 7^{-1} + 2 \times 7^{-2} + 1 \times 7^{-3}\]

(2.50) Convert to decimal form.

(a) \[4205_6 = 4 \times 6^3 + 2 \times 6^2 + 5 = 941_{10}\]

(b) \[142032_5 = 1 \times 5^5 + 4 \times 5^4 + 2 \times 5^3 + 3 \times 5 + 2 = 5892_{10}\]

(2.51) Convert to decimal form.

(a) \[24.042_5 = 2 \times 5 + 4 + 4 \times 5^{-2} + 2 \times 5^{-3} = 14.176_{10}\]

(b) \[2.13_4 = 2 + 1 \times 4^{-1} + 3 \times 4^{-2} = 2.4375_{10}\]

(2.52) Rewrite the decimal number 3263 to the base:

(a) 5

\[3263 = 5 \cdot 652 + 3\]
\[652 = 5 \cdot 130 + 2\]
\[130 = 5 \cdot 26 + 0\]
\[26 = 5 \cdot 5 + 1\]
\[5 = 5 \cdot 1 + 0\]
\[1 = 5 \cdot 0 + 1\]

\[3263_{10} = 101023_5\]
2.1 Number systems

(b) 4

\begin{align*}
3263 &= 4 \cdot 815 + 3 \\
815 &= 4 \cdot 203 + 3 \\
203 &= 4 \cdot 50 + 3 \\
50 &= 4 \cdot 12 + 2 \\
12 &= 4 \cdot 3 + 0 \\
3 &= 4 \cdot 0 + 3 \\
\end{align*}

$3263_{10} = 302333_4$

(c) 12 (using A=10 and B=11)

\begin{align*}
3263 &= 12 \cdot 271 + 11 \\
271 &= 12 \cdot 22 + 7 \\
22 &= 12 \cdot 1 + 10 \\
1 &= 12 \cdot 0 + 1 \\
\end{align*}

$3263_{10} = 1A7B_4$

(2.53) Rewrite the decimal number 1547 to the base:

(a) 6

\begin{align*}
1547 &= 6 \cdot 257 + 5 \\
257 &= 6 \cdot 42 + 5 \\
42 &= 6 \cdot 7 + 0 \\
7 &= 6 \cdot 1 + 1 \\
1 &= 6 \cdot 0 + 1 \\
\end{align*}

$1547_{10} = 11055_6$

(b) 9

\begin{align*}
1547 &= 9 \cdot 171 + 8 \\
171 &= 9 \cdot 19 + 0 \\
19 &= 9 \cdot 2 + 1 \\
2 &= 9 \cdot 0 + 2 \\
\end{align*}

$1547_{10} = 2108_9$

(c) 12 (using A=10 and B=11)

\begin{align*}
1547 &= 12 \cdot 128 + 11 \\
128 &= 12 \cdot 10 + 8 \\
10 &= 12 \cdot 0 + 10 \\
\end{align*}

$1547_{10} = A8B_{12}$
2.1 Number systems

(2.54) Convert the decimal number 274.824 to its base-5 form. First we convert the integral part \( N_i = 274 \) by dividing.

\[
\begin{align*}
274 &= 5 \cdot 54 + 4 \\
54 &= 5 \cdot 10 + 4 \\
10 &= 5 \cdot 2 + 0 \\
2 &= 5 \cdot 0 + 2
\end{align*}
\]

Next we convert the fractional part \( N_f = 0.824 \) by multiplying.

\[
\begin{align*}
0.824 \cdot 5 &= 4.12 \\
0.12 \cdot 5 &= 0.6 \\
0.6 \cdot 5 &= 3.0
\end{align*}
\]

Now the base-5 equivalent is the sum of two pieces we calculated above.
\[
274.824_{10} = 2044_{5} + 0.403_{5} = 2044.403_{5}
\]

(2.55) Convert the decimal number 145.6875 to its base-4 form. First we convert the integral part \( N_i = 145 \) by dividing.

\[
\begin{align*}
145 &= 4 \cdot 36 + 1 \\
36 &= 4 \cdot 9 + 0 \\
9 &= 4 \cdot 2 + 1 \\
2 &= 4 \cdot 0 + 2
\end{align*}
\]

Next we convert the fractional part \( N_f = 0.6875 \) by multiplying.

\[
\begin{align*}
0.6875 \cdot 4 &= 2.75 \\
0.75 \cdot 4 &= 3.0
\end{align*}
\]

Now the base-4 equivalent is the sum of the two pieces we calculated above.
\[
145.6875_{10} = 2101_{4} + 0.23_{4} = 2101.23_{4}
\]

(2.56) Convert the decimal number 0.3 to its base-4 form.

\[
\begin{align*}
0.3 \cdot 4 &= 1.2 \\
0.2 \cdot 4 &= 0.8 \\
0.8 \cdot 4 &= 3.2 \\
0.2 \cdot 4 &= 0.8
\end{align*}
\]

\[
0.3_{10} = 0.10303030\ldots_{4}
\]
2.2 Octal System

(2.57) Convert each number to its octal form:

(a) 12345

\[
12345 = 8 \cdot 1543 + 1 \\
1543 = 8 \cdot 192 + 7 \\
192 = 8 \cdot 24 + 0 \\
24 = 8 \cdot 3 + 0 \\
3 = 8 \cdot 0 + 3
\]

\[12345_{10} = 30071_8\]

(b) 44444

\[
44444 = 8 \cdot 5555 + 4 \\
5555 = 8 \cdot 694 + 3 \\
694 = 8 \cdot 86 + 6 \\
86 = 8 \cdot 10 + 6 \\
10 = 8 \cdot 1 + 2 \\
1 = 8 \cdot 0 + 1
\]

\[44444_{10} = 126634_8\]

(2.58) Convert to decimal form:

(a) 12345\textsubscript{8}

\[
12345\textsubscript{8} = 1 \cdot 8^4 + 2 \cdot 8^3 + 3 \cdot 8^2 + 4 \cdot 8^1 + 5 \cdot 8^0 \\
= 5394\textsubscript{10}
\]

(b) 44444\textsubscript{8}

\[
44444\textsubscript{8} = 4 \cdot 8^4 + 4 \cdot 8^3 + 4 \cdot 8^2 + 4 \cdot 8^1 + 4 \cdot 8^0 \\
= 18724\textsubscript{10}
\]

(2.59) Convert each decimal number to its octal form:

(a) 0.4375

\[
0.4375 \cdot 8 = 3 \quad .5 \\
0.5 \cdot 8 = 4 \quad .0
\]

\[0.4375\textsubscript{10} = 0.34\textsubscript{8}\]
2.2 Octal System

(b) 0.4

\[
\begin{align*}
0.4 \cdot 8 &= 3 \cdot 2 \\
0.2 \cdot 8 &= 1 \cdot 6 \\
0.6 \cdot 8 &= 4 \cdot 8 \\
0.8 \cdot 8 &= 6 \cdot 4 \\
0.4 \cdot 8 &= 3 \cdot 2
\end{align*}
\]

0.4\textsubscript{10} = 0.3146\textsubscript{3146} \ldots 8

(2.60) Convert to binary form:

(a) 617025\textsubscript{8}

We convert each digit to its binary equivalent:

617025\textsubscript{8} = 110 001 111 000 010 101\textsubscript{2}

(b) 43.0276\textsubscript{8}

43.0276\textsubscript{8} = 100 011.000 010 111 110\textsubscript{2}

(2.61) Convert to octal form:

(a) 10101111100\textsubscript{2}

We split this number in groups of three digits starting from the right and convert each part:

010 101 111 100\textsubscript{2} = 2574\textsubscript{8}

(b) 1000110111\textsubscript{2} 001 000 110 111\textsubscript{2} = 1067\textsubscript{8}

(c) 1011.01011\textsubscript{2} 001 011.010 110\textsubscript{2} = 13.26\textsubscript{8}

(2.62) Add the following octal digits:

- 4 + 3 = 7
- 5 + 6 = 13
- 2 + 4 = 6
- 6 + 7 = 15
- 7 + 4 = 13
- 1 + 4 = 5
- 3 + 6 = 11
- 4 + 6 = 12
- 7 + 7 = 16
2.2 Octal System

(2.63) Evaluate:

(a) 45376₈ + 36274₈

```
  45376
+ 36274
  +-----
  103672
```

(b) 257365₄₈ + 44477₇₈

```
  257365₈
+ 44477₇₈
  +-------
  324063₈
```

(c) 333.5₆₇₈ + 47.₄₇₄₇₈

```
  333.₅₆₇₀₈
+ 47.₄₇₄₇₈
  +-------
  403.₂₆₃₇₈
```

(2.64) Find the radix-minus-one (7s) complement and the (8s) complement of:

(a) 23470₅₈

```
7s complement: 54307₂₈
8s complement: 54307₃₈
```

(b) 11335₅₈

```
7s complement: 66442₂₈
8s complement: 66442₃₈
```

(c) 6660₀₈

```
7s complement: 1117₇₈
8s complement: 1120₀₈
```

(2.65) Evaluate, using complements:

Perform the subtraction by adding the complement of the subtrahend to the minuend.
2.3 Hexadecimal System

(a) $6157_8 - 4325_8$

\[
\begin{align*}
\text{Minuend:} & \quad 6157 \\
\text{Complement of subtrahend:} & \quad 3452 \\
\text{Replace the leading one:} & \quad 11631 \\
\text{Result:} & \quad 1632
\end{align*}
\]

(b) $671354_8 - 213604_8$

\[
\begin{align*}
\text{Minuend:} & \quad 671354 \\
\text{Complement of subtrahend:} & \quad 564173 \\
\text{Replace the leading one:} & \quad 1455547 \\
\text{Result:} & \quad 455550
\end{align*}
\]

2.3 Hexadecimal System

(2.66) Convert each decimal number to its hexadecimal form:

(a) 967

\[
\begin{align*}
967 & = 16 \cdot 60 + 7 \\
60 & = 16 \cdot 3 + C \\
3 & = 16 \cdot 0 + 3
\end{align*}
\]

$967_{10} = 3C7_{16}$

(b) 2893

\[
\begin{align*}
2893 & = 16 \cdot 180 + D \\
180 & = 16 \cdot 11 + 4 \\
11 & = 16 \cdot 0 + B
\end{align*}
\]

$2893_{10} = B4D_{16}$

(2.67) Convert to decimal form:

(a) $3E7_{16}$

\[
\begin{align*}
3E7_{16} & = 3 \cdot 16^2 + 14 \cdot 16 + 7 \\
& = 999_{10}
\end{align*}
\]
2.3 Hexadecimal System

(b) \(4A5C_{16}\)

\[4A5C_{16} = 4 \cdot 16^3 + 10 \cdot 16^2 + 5 \cdot 16 + 12\]
\[= 19036_{10}\]

(2.68) Convert the decimal fraction 0.3 to its hexadecimal form.

\[0.3 \cdot 16 = 4.8\]
\[0.8 \cdot 16 = 12.8\]
\[0.8 \cdot 16 = 12.8\]

\[0.3_{10} = 0.4CC\ldots\]

(2.69) Convert to binary form:

(a) \(B9E4_{16}\)
\[B9E4_{16} = 1011 1001 1110 0100_2\]

(b) \(50C7F6_{16}\)
\[50C7F6_{16} = 0101 0000 1100 0111 1111 0110_2\]

(2.70) Convert to hexadecimal form:

(a) \(11101101101100_2\)
\[0011 1011 0110 1100_2 = 3B6C_{16}\]

(b) \(111000111110_2\)
\[0001 1100 0111 1110 = 1C7E_{16}\]

(c) \(111110.1011111_2\)
\[0011 1110.1011 1100_2 = 3E.BC_{16}\]

(2.71) Add the following hexadecimal digits:

- \(5 + 7 = C\)
- \(9 + 8 = 11\)
- \(B + 2 = D\)
- \(7 + 3 = A\)
- \(2 + 4 = 6\)
- \(E + E = 1C\)
- \(6 + A = 10\)
- \(C + 6 = 12\)
- \(4 + 9 = D\)

(2.72) Evaluate:

(a) \(47B6_{16} + 9C75_{16}\)
2.3 Hexadecimal System

\[ \begin{array}{c}
47B6 \\
9C75 \\
+____ \\
E42B \\
\end{array} \]

(b) \[ \begin{array}{c}
8D07A5_{16} + 734F6_{16} \\
8D07A5 \\
734F6 \\
+_______ \\
943C9B \\
\end{array} \]

(c) \[ \begin{array}{c}
67.E9_{16} + A.BCDE_{16} \\
67.E9 \\
A.BCDE \\
+_______ \\
72.A5DE \\
\end{array} \]

(2.73) Find the radix-minus-one (15s) complement and the (16s) complement of:

(a) \[ \begin{array}{c}
5D309_{16} \\
15s \ A2CF6_{16} \\
16s \ A2CF7_{16} \\
\end{array} \]

(b) \[ \begin{array}{c}
2A4E61_{16} \\
15s \ D5B19E_{16} \\
16s \ D5B19F_{16} \\
\end{array} \]

(c) \[ \begin{array}{c}
A1B2C300_{16} \\
15s \ 4E4D3CFF_{16} \\
16s \ 4E4D3D00_{16} \\
\end{array} \]

(2.74) Evaluate, using complements:

We perform the subtraction by adding the complement of the subtrahend to the minued. Then we subtract the leading zero and add it to the least significant digit (the most right).
2.4 4-bit BCD codes

(a) $76B_{16} - 432C_{16}$

Minuend: $76B$
Complement of subtrahend: BCD3

\[
\begin{array}{c}
\text{Replace the leading 1:} \\
\rightarrow 1
\end{array}
\]

Result: 3389

(b) $A57913_{16} - 64EE00_{16}$

Minuend: $A57913$
Complement of subtrahend: 9B11FF

\[
\begin{array}{c}
\text{Replace the leading 1:} \\
\rightarrow 1
\end{array}
\]

Result: 408B13

2.4 4-bit BCD codes

(2.75) Decode each numeric, expressed in the 8-4-2-1 BCD code:

(a) 0110 1001 0111

\[
\begin{align*}
0110 & \quad 1001 & \quad 0111 & \quad = (4 + 2) \cdot 10^2 + (8 + 1) \cdot 10 + 7 \\
& \quad & \quad & \quad = 697
\end{align*}
\]

(b) 0011 0100 1000 0101

\[
\begin{align*}
0011 & \quad 0100 & \quad 1000 & \quad 0101 & \quad = (2 + 1) \cdot 10^3 + (4) \cdot 10^2 + 8 \cdot 10 + (4 + 1) \\
& \quad & \quad & \quad & \quad = 3485
\end{align*}
\]

(2.76) Decode each numeric, encoded in the XS-3 BCD code:

The XS-3 code for a decimal digit is obtained by $3(=0011_2)$ to the BCD code.

(a) 0101 1011 1000

\[
\begin{align*}
0101 & \quad 1011 & \quad 1000 & \quad = (5 - 3) \cdot 10^2 + (11 - 3) \cdot 10 + (8 - 3) \\
& \quad & \quad & \quad = 285
\end{align*}
\]
2.4 4-bit BCD codes

(b) 0111 1100 0011 0100 1010

\[
0111 \quad 1100 \quad 0011 \quad 0100 \quad 1010 = (7 - 3) \cdot 10^4 \\
+ (12 - 3) \cdot 10^3 \\
+ (3 - 3) \cdot 10^2 \\
+ (4 - 3) \cdot 10 \\
+ (10 - 3) \\
= 49017
\]

(2.77) Decode each numeric, encoded in the 5-4-2-1 BCD code:

(a) 1010 0010 1001

\[
1010 \quad 0010 \quad 1001 = 726
\]

(b) 1011 0001 0100 1100

\[
1011 \quad 0001 \quad 0100 \quad 1100 = 8149
\]

(2.78) Encode each decimal number in the 8-4-2-1 BCD code:

(a) 395

\[
395 = 0011 \quad 1001 \quad 0101
\]

(b) 70246

\[
70246 = 0111 \quad 0000 \quad 0010 \quad 0100 \quad 0110
\]

(2.79) Encode each decimal number in the XS-3 BCD code:

We use the solutions of 2.78 and add 0011\textsubscript{2} to each digit.

(a) 395

\[
395 = 0011 \quad 1001 \quad 0101 \\
= 0110 \quad 1100 \quad 1000
\]

(b) 70246

\[
70246 = 0111 \quad 0000 \quad 0010 \quad 0100 \quad 0110 \\
= 1010 \quad 0011 \quad 0101 \quad 0111 \quad 1001
\]

(2.80) Given that 0110 0011 1001 1011 is the XS-3 code for the decimal A, find the XS-3 code for the (10s) complement of A, without decoding A.

elie@de-brauwer.be
The XS-3 has an important arithmetic property: it encodes a pair of nines complements as a pair of ones complements. In this case we may simply take the 1s complement (9s complement) of the given XS-3 code and add 1 to obtain the 2s complement (10s complement).

<table>
<thead>
<tr>
<th>XS-3 code</th>
<th>0110 0011 1001 1011</th>
</tr>
</thead>
<tbody>
<tr>
<td>1s complement:</td>
<td>1001 1100 0110 0100</td>
</tr>
<tr>
<td>2s complement:</td>
<td>1001 1100 0110 0101</td>
</tr>
</tbody>
</table>

2.5 6-bit BCD codes

(2.81) Encode the decimal number 4839 in the 6-bit BCD system (without using a table) in:

- Digits are coded with 0s for both zone bits and their 8-4-2-1 BCD code for the numeric bits.
  - (a) binary form
    
    \[
    \begin{align*}
    000100 & \quad 001000 & \quad 000011 & \quad 001001 \\
    \end{align*}
    \]
  - (b) octal form
    
    04 10 03 11

(2.82) What is the minimum number of character blocks required to encode the message:

IN THE BEGINNING

16, 14 alphabetical character and 2 whitespaces.

(2.83) Suppose that a computer uses the 6-bit BCD code, with odd parity. How would the computer store the name:

For each character, the value of the check bit (0 or 1) is such as to make the sum of the bits, including the check bit, odd or even, according as the machine operates on odd or even parity.

(a) MARC

<table>
<thead>
<tr>
<th>Character</th>
<th>Check bit</th>
<th>BCD code</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>1</td>
<td>100100</td>
</tr>
<tr>
<td>A</td>
<td>0</td>
<td>110001</td>
</tr>
<tr>
<td>R</td>
<td>0</td>
<td>101001</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>110011</td>
</tr>
</tbody>
</table>
(2.84) Suppose that a computer uses even parity and that the data item HAMLET is stored in the computer as follows:

\[ \begin{array}{cccc}
1111000 & 1110001 & 0100101 & 1100011
\end{array} \]

Without using a table find which letters, if any, contain an error. The checkbit of the third letter is not correct, so the letter M contains an error.

2.6 8-bit BCD codes

(2.85) Encode the name AUDREY in:

(a) binary EBCDIC

(b) hexadecimal EBCDIC

(c) How would the binary code appear in the computer if the computer uses odd parity checking.

<table>
<thead>
<tr>
<th>Character</th>
<th>EBCDIC</th>
<th>EBCDIC-HEX</th>
<th>Checksum</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>11000001</td>
<td>C1</td>
<td>0</td>
</tr>
<tr>
<td>U</td>
<td>11100100</td>
<td>E4</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>11000100</td>
<td>C4</td>
<td>0</td>
</tr>
<tr>
<td>R</td>
<td>11011001</td>
<td>D9</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>11000101</td>
<td>C5</td>
<td>1</td>
</tr>
<tr>
<td>Y</td>
<td>11101000</td>
<td>E8</td>
<td>1</td>
</tr>
</tbody>
</table>

(2.86) Repeat problem 2.85 using ASCII-8 instead of EBCDIC.

<table>
<thead>
<tr>
<th>Character</th>
<th>ASCII</th>
<th>ASCII-HEX</th>
<th>Checksum</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10100001</td>
<td>A1</td>
<td>0</td>
</tr>
<tr>
<td>U</td>
<td>10110101</td>
<td>B5</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>10100100</td>
<td>A4</td>
<td>0</td>
</tr>
<tr>
<td>R</td>
<td>10110010</td>
<td>B2</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>10100101</td>
<td>A5</td>
<td>1</td>
</tr>
<tr>
<td>Y</td>
<td>10111001</td>
<td>B9</td>
<td>0</td>
</tr>
</tbody>
</table>

(2.87) Using zoned decimal format, write the EBCDIC codes for:

(a) \(+3759 = 111100011111011111110110111001001\)

(b) \(-3759 = 11110001111110111111101011011011001\)

(c) \(3759 = 111100011111101111101010111111\)
(2.88) Using packed decimal format, write the codes for:

(a) \(+3759 = 00110111010110011100\)
(b) \(-3759 = 00110111010110011101\)
(c) \(3759 = 00110111010110011111\)
### 3 Computer Arithmetic

#### 3.1 Mathematical Preliminaries

(3.21) Determine the most significant digit, the least significant digit, and the number of significant digits in:

<table>
<thead>
<tr>
<th>Digit</th>
<th>Most significant</th>
<th>Least significant</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>44.44</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>30303</td>
<td>3</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>6.6707</td>
<td>6</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>5.005</td>
<td>5</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>0.222</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>0.0003333</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>0.0011</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>0.008008</td>
<td>8</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>2.22000</td>
<td>2</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>3300.000</td>
<td>3</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>0.0044400</td>
<td>4</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>5550000</td>
<td>5</td>
<td>0</td>
<td>6</td>
</tr>
</tbody>
</table>

(3.22) Round 22.4444, 1.234567, 333.777, 0.06543211, 0.005678

<table>
<thead>
<tr>
<th>Number</th>
<th>2 decimal places</th>
<th>3 decimal places</th>
<th>4 sign. digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.4444</td>
<td>22.44</td>
<td>22.444</td>
<td>22.44</td>
</tr>
<tr>
<td>1.234567</td>
<td>1.23</td>
<td>1.235</td>
<td>1.235</td>
</tr>
<tr>
<td>333.777</td>
<td>333.78</td>
<td>333.777</td>
<td>333.8</td>
</tr>
<tr>
<td>0.0654321</td>
<td>0.07</td>
<td>0.065</td>
<td>0.06543</td>
</tr>
<tr>
<td>0.005678</td>
<td>0.01</td>
<td>0.006</td>
<td>0.005678</td>
</tr>
</tbody>
</table>

(3.23) Round 0.44500, 7.775, 66.665000, 8.885020, 2.3350

<table>
<thead>
<tr>
<th>Digit</th>
<th>2 decimal places</th>
<th>2 significant digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.44500</td>
<td>0.44</td>
<td>0.44</td>
</tr>
<tr>
<td>7.775</td>
<td>7.78</td>
<td>7.8</td>
</tr>
<tr>
<td>66.66500</td>
<td>66.66</td>
<td>67</td>
</tr>
<tr>
<td>8.885020</td>
<td>8.89</td>
<td>8.9</td>
</tr>
<tr>
<td>2.3350</td>
<td>2.34</td>
<td>2.3</td>
</tr>
</tbody>
</table>

(3.24) Truncate 44.44, 30303, 6.6707, 5.005, -0.0444888

<table>
<thead>
<tr>
<th>Digit</th>
<th>Integer</th>
<th>4 significant digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>44.44</td>
<td>44</td>
<td>44.44</td>
</tr>
<tr>
<td>30303</td>
<td>30300</td>
<td>30300</td>
</tr>
<tr>
<td>6.6707</td>
<td>6</td>
<td>6.670</td>
</tr>
<tr>
<td>5.005</td>
<td>5</td>
<td>5.005</td>
</tr>
<tr>
<td>-0.0444888</td>
<td>0</td>
<td>-0.04448</td>
</tr>
</tbody>
</table>

(3.25) Let CHOP(Q) denote the integral part of the number Q. Solve the
3.2 Exponential Form

\[ 2 \cdot \text{CHOP}\left(\frac{N}{2}\right) = N^1. \]

(3.26) Evaluate
\[
\begin{align*}
(a) & \quad |4 - 9| = |-5| = 5 \\
(b) & \quad |-4 - 9| = |-13| = 13 \\
(c) & \quad |-4 + 9| = |5| = 5 \\
(d) & \quad |3 - 5| - |6 - 2| = 2 - 4 = -2 \\
(e) & \quad |\|-6| - |3 - 12|| = |6 - 9| = 3
\end{align*}
\]

3.2 Exponential Form

(3.27) Rewrite each number without an exponent:
\[
\begin{align*}
(a) & \quad 44.44 \cdot 10^4 = 444400 \\
(b) & \quad 55.55 \cdot 10^{-5} = 0.000555 \\
(c) & \quad -0.066 \cdot 10^2 = -6.6 \\
(d) & \quad 0.0077 \cdot 10^{-3} = 0.0000077 \\
(e) & \quad 88.99 \cdot 10^0 = 88.99 \\
(f) & \quad 1.234 \cdot 10^{-4} = 0.0001234
\end{align*}
\]

(3.28) Write each number in normalized\(^2\) exponential form:
\[
\begin{align*}
(a) & \quad 333.444 = 0.333444 \cdot 10^3 \\
(b) & \quad -1.2345 = -0.12345 \cdot 10^1 \\
(c) & \quad 0.0006677 = 0.6677 \cdot 10^{-3} \\
(d) & \quad -0.8899 = -0.8899 \cdot 10^0 \\
(e) & \quad 222000 = 0.222 \cdot 10^6 \\
(f) & \quad -0.03 = -0.3 \cdot 10^{-1}
\end{align*}
\]

(3.29) Find the mantissa and exponent of each number in Problem 3.28.

<table>
<thead>
<tr>
<th>Number</th>
<th>Mantissa</th>
<th>Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>333.444</td>
<td>0.333444</td>
<td>4</td>
</tr>
<tr>
<td>-1.2345</td>
<td>-0.12345</td>
<td>1</td>
</tr>
<tr>
<td>0.0006677</td>
<td>0.6677</td>
<td>-3</td>
</tr>
<tr>
<td>-0.8899</td>
<td>-0.8899</td>
<td>0</td>
</tr>
<tr>
<td>222000</td>
<td>0.222</td>
<td>6</td>
</tr>
<tr>
<td>-0.03</td>
<td>-0.3</td>
<td>-1</td>
</tr>
</tbody>
</table>

\(^1\)If someone is able to point me to the right solution of this questions I’d appreciate it, I have trouble understanding this question since English isn’t my mother tongue. Mail to elie@de-brauwer.be

\(^2\)A = M \cdot 10^x; -1 < M < 1
3.3 Internal Representation

(3.30) Write each number in scientific notation:

(a) \(333.444 = 3.33444 \cdot 10^2\)
(b) \(-1.2345 = -1.2345 \cdot 10^0\)
(c) \(0.000677 = 6.77 \cdot 10^{-4}\)
(d) \(-0.8899 = -8.899 \cdot 10^{-1}\)
(e) \(222000 = 2.22 \cdot 10^5\)
(f) \(-0.03 = -3 \cdot 10^{-2}\)

(3.31) Find the value of each number written in computer E-form

(a) \(222.2E + 2 = 222.2 \cdot 10^2 = 22220\)
(b) \(-3.3E - 03 = -3.3 \cdot 10^{-3} = -0.0033\)
(c) \(0.4E4 = 0.4 \cdot 10^4 = 4000\)
(d) \(0.5E - 5 = 0.5 \cdot 10^{-5} = 0.00005\)

(3.32) Give the binary normalized form of each number:

(a) \(111.000111 = 0.111000111 \cdot 2^3\)
(b) \(0.0011001100 = 0.11001100 \cdot 2^{-2}\)
(c) \(-1010.1010 = -0.10101010 \cdot 2^4\)
(d) \(0.1111 = 0.1111 \cdot 2^0\)

3.3 Internal Representation

In Problems 3.33 through 3.38, assume that the computer stores each number in a 32-bit memory location.

(3.33) Find the internal representation of:

(a) 118

\(118_{10} = 11101102\) but since this number has to be stored within a 32-bit memory location we have to add 0’s to make the number 32 bit long.

(b) -118

This is the two’s complement of the prior solution 111...0001010.

(3.34) Find the internal representation of:

(a) 397

\(397_{10} = 1100011012\) and add 0’s to make this a 32 bit long number.

---

\(^3A = M \cdot 10^n; -10 < M < 10\)

\(^4A = M \cdot 2^n; M \) starts with \(\pm 0.1\ldots\)
3.4 Computer Arithmetic

(b) -397 The solution is given by the two’s complement of the solution of problem 3.34a 111...001110011.

(3.35) Find the internal representation of $A = -50.375$
Since this is a floating point number the first bit is used to store the sign, the second 7 bits are used to store the characteristic $^5$ and the last 24 bits are used to store the mantissa. The mantissa and the exponent are obtained by using the binary normalized form.
The Integral part:
$50_{10} = 110010_2$
The Fractional part:
$0.375_{10} = 0.011_2$
Thus $A = -110010.011 \cdot 2^0 = -0.110010010 \cdot 2^6$, when we want to store this into a 32-bit memory location we obtain the value : $11001101100100100000000000000000_2$. $^6$

(3.36) Find the internal representation of $B = 0.09375$
The Integral part is nonexistent so we start with the fractional part:
$0.09375_{10} = 0.00111_2$
Thus $A = 0.00011 = 0.11 \cdot 2^{-3}$ this results in an internal representation given by $0011111000000000000000000_2$.

(3.37) Find the internal representation of $C = 0.2$ The fractional part:
$0.2_{10} = 0.00110110011\ldots$
Thus $A = 0.00110011\ldots = 0.1100\ldots \cdot 2^{-2}$, which results in an internal representation given by $0011111010010011001001100_2$.

(3.38) Suppose that an exponent $n$ is represented in a 7-bit field as follows.
The first bit is reserved for the sign, 1 for + and 0 for -. In the remaining 6-bit field, $n$ is represented as its binary form if $n$ is positive, but as its 2s complement if $n$ is negative. Show that this is exactly the same representation as given bij storing the 7-bit characteristics, $C = n + 64$, of $n$.
This can be accomplished by simply writing out the numbers.

3.4 Computer Arithmetic

In Problems 3.41 through 3.44, assume that the computer truncates mantissas to $P = 4$ decimal digits.

(3.39) If the computer is programmed to perform fixed-point integer arithmetic$^7$, what values are obtained for:

$^5$Characteristic = exponent + 64
$^6$In this case the characteristic is $64 + 6 = 70_{10} = 1000110_2$
$^7$The main property of integer arithmetic is that the result is integer
3.4 Computer Arithmetic

(a) \(6 + 10, 2 - 7, 3 \times (-5), -4 - 8, -4 - (6 - 3)?\)
\[- 6 + 10 = 16\]
\[- 2 - 7 = -5\]
\[- 3 \times (-5) = -15\]
\[- 4 - 8 = -12\]
\[- 4 - (6 - 3) = -7\]

(b) \(\frac{11}{4}, -\frac{15}{3}, \frac{123}{4}, -\frac{26}{8}, -\frac{5}{8}?\)
\[- \frac{11}{4} = 2\]
\[- \frac{15}{3} = -5\]
\[- \frac{8}{11} = 0\]
\[- \frac{123}{4} = 30\]
\[- \frac{26}{8} = -3\]
\[- \frac{5}{8} = 0\]

3.40 Under integer arithmetic, the quotient \(\frac{J}{K}\) of two integers \(J\) and \(K\) is less than or equal to the usual quotient. (True or false).
This is only true if the result is larger than zero, \(\frac{-33}{8}\) equals 4.125 which is greater than 4. When we look at \(-\frac{33}{8}\) the result is \(-4.125\) which is lesser than \(-4\). So this statement is false.

3.41 Give the results of the three floating-point additions:\(^5\)
(a) \(0.2233 \cdot 10^2 + 0.6688 \cdot 10^1 = 0.2901 \cdot 10^2\)
(b) \(5.666 \cdot 10^1 + 44.55 = 0.5021 \cdot 10^2\)
(c) \(9 \cdot 111.77 + 55.666 = 0.1674 \cdot 10^3\)

3.42 Perform the following floating-point subtractions:\(^10\)
(a) \(0.9922 \cdot 10^{-3} - 0.4477 \cdot 10^{-3} = 0.5452 \cdot 10^{-3}\)
(b) \(33.666 - 2.7777 = 0.3088 \cdot 10^2\)
(c) \(0.888 \cdot 10^2 - 0.2222 \cdot 10^3 = -0.1334 \cdot 10^3\)

3.43 Give the results of the following floating-points multiplications:\(^11\)
(a) \((0.5432 \cdot 10^3) \cdot (0.3333 \cdot 10^{-5}) = 0.1810 \cdot 10^{-2}\)
(b) \(222.88 \cdot 1.1177 = 0.2488 \cdot 10^3\)

\(^5\)If two numbers to be added have the same exponent, the mantissas are added and the same exponent is used.
\(^9\)In the book the result is 0.674 \cdot 10^2, this is obviously a mistake in the book.
\(^10\)Analogous to real addition.
\(^11\)Here we multiply the mantissas and add the exponents.
3.5 Errors

(3.44) Perform the following floating-point divisions:
(a) \( \frac{0.2233 \times 10^{-2}}{0.6611 \times 10^{-3}} = 0.3377 \times 10^{-5} \)
(b) \( \frac{111.99}{41.888} = 2.493 \times 10^{1} \)

3.5 Errors

(3.45) Given \( A = 66.888 \), find the absolute error \( e \) and the relative error \( r \) if
(a) \( A \) is rounded to 66.89
\[ e = 66.888 - 66.89 = -0.002 \]
\[ r = \frac{-0.002}{66.89} = -0.002989983\% \]
(b) \( A \) is truncated to 66.88
\[ e = 66.888 - 66.88 = 0.008 \]
\[ r = \frac{0.008}{66.88} = 0.01196\% \]

(3.46) Given \( A = 66.888 \) and \( B = 66.111 \). When the computer calculates the difference \( D = A - B \) (where mantissas are truncated to \( P = 4 \) digits), what are the absolute and relative errors?
With truncating:
\[ D = A - B \]
\[ D = 66.88 - 66.11 \]
\[ D = 0.77 \]

Without truncating:
\[ D = A - B \]
\[ D = 66.888 - 66.111 \]
\[ D = 0.777 \]

The absolute error is given by:
\[ e = 0.777 - 0.77 = 0.007 \]

The relative error is given by:
\[ r = \frac{0.007}{0.77} = 0.9091\% \]

(3.47) Place a bound on the absolute error when two approximate numbers are added.
\[ A \pm e_a + B \pm e_b = (A + B) \pm (e_a \pm e_b) \]
\[ |e_{a+b}| = |e_a + e_b| \]
\[ |e_{a+b}| \leq |e_a| + |e_b| \]

---

12Here we divide the mantissas and subtract the exponents. The division is carried out only to \( P \) significant digits

elie@de-brauwer.be 34
4 Logic, Truth Tables

4.1 Statements and compound statements

(4.23) Let p be *Marc is rich* and let q be *Marc is happy*. Write each of the following in symbolic form:\(^{13}\)

(a) Marc is poor but happy: \(\neg P \land Q\)

(b) Marc is neither rich nor happy: \(P \land \neg Q\)

(c) Marc is either rich or unhappy: \(P \lor \neg Q\)

(d) Marc is poor or else he is both rich and unhappy: \(\neg P \lor (P \land \neg Q)\)

(4.24) Let p be *Erik reads Newsweek*, let q be *Erik reads The New Yorker* and let r be *Erik reads Time*. Write each of the following statements in symbolic form:

(a) Erik reads Newsweek or The New Yorker but not Time: \((P \lor Q) \land \neg R\)

(b) Erik reads Newsweek and The New Yorker, or he does not read Newsweek and Time \((P \land Q) \lor (P \land \neg R)\)

(c) It is not true that Erik reads Newsweek but not Time: \(P \land \neg R\)

(d) It is not true that Erik reads Time or The New Yorker but not Newsweek \((Q \lor R) \land \neg P\)

(4.25) Let p be *Audrey speaks French* and let q be *Audrey speaks Danish*. Give a simple verbal sentence which describes each of the following.

(a) \(P \lor Q\) Audrey speaks French or Danish.
(b) \(P \land Q\) Audrey speaks French and Danish.
(c) \(P \land \neg Q\) Audrey speaks French but not Danish.
(d) \(\neg P \lor \neg Q\) Audrey doesn’t speak French or she doesn’t speak Danish.
(e) \(\neg P\) Audrey speaks French.
(f) \(\neg P \land \neg Q\) Audrey speaks French and Danish\(^{14}\)

\(^{13}\)Please note that I use \(\neg A\) instead of \(\neg\)\(A\)
\(^{14}\)Using DeMorgan’s law
4.2 Truth tables, logical equivalence

(4.26) Find the truth table of each proposition:

(a) $P \lor \overline{Q}$

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>$\overline{P}$</th>
<th>$\lor$</th>
<th>$\overline{Q}$</th>
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<tbody>
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(b) $\overline{P} \land \overline{Q}$

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(c) $\overline{P} \land Q$

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(d) $\overline{P} \lor \overline{Q}$

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(4.27) Find the truth table of each proposition:

(a) $(P \land \overline{Q}) \lor R$
4.2 Truth tables, logical equivalence

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>P ( \land ) Q</th>
<th>Q ( \lor ) R</th>
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(b) \( P \lor (Q \land R) \)

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(c) \( (P \lor \overline{R}) \land (Q \lor \overline{R}) \)

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(4.28) Prove that conjunction distributes over disjunction \( P \land (Q \lor R) \iff (P \land Q) \lor (P \land R) \).

(4.29) Prove \( P \lor (P \land Q) \iff P \) by constructing the the appropriate truth tables.

(4.30) Prove \( (P \lor Q) \lor (P \land Q) \iff \overline{P} \) by constructing the the appropriate truth tables.

Problems 4.28 thru 4.30 are not solved here because solving these
4.2 Truth tables, logical equivalence

problems is nothing more than writing the truth tables. All these truth tables result in tautologies because the problems are nothing more than the Laws of the Algebra of Propositions. See problem 4.32 for more information about these laws.

(4.31) (a) Express $\vee$ in terms of $\land$ and $\neg$.

$$A \lor B \iff \overline{A \land B}$$

(b) Express $\lor$ in terms of $\land$ and $\neg$.

$$A \lor B \iff \overline{A \lor B}$$

(4.32) Prove the following equivalences by using the laws of algebra of propositions listed below.\(^{15}\)

<table>
<thead>
<tr>
<th>Laws of the Algebra of Propositions</th>
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<tbody>
<tr>
<td><strong>Idempotent Laws</strong></td>
</tr>
<tr>
<td>$P \lor P \iff P$</td>
</tr>
<tr>
<td>$P \land P \iff P$</td>
</tr>
<tr>
<td><strong>Associative Laws</strong></td>
</tr>
<tr>
<td>$(P \lor Q) \lor R \iff P \lor (Q \lor R)$</td>
</tr>
<tr>
<td>$(P \land Q) \land R \iff P \land (Q \land R)$</td>
</tr>
<tr>
<td><strong>Commutative Laws</strong></td>
</tr>
<tr>
<td>$P \lor Q \iff Q \lor P$</td>
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<tr>
<td>$P \land Q \iff Q \land P$</td>
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<tr>
<td><strong>Distributive Laws</strong></td>
</tr>
<tr>
<td>$P \lor (Q \land R) \iff (P \lor Q) \land (P \lor R)$</td>
</tr>
<tr>
<td>$P \land (Q \lor R) \iff (P \land Q) \lor (P \land R)$</td>
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<tr>
<td><strong>Identity Laws</strong></td>
</tr>
<tr>
<td>$P \lor F \iff P$</td>
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<tr>
<td>$P \land T \iff P$</td>
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<tr>
<td>$P \lor T \iff T$</td>
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<tr>
<td>$P \land F \iff P$</td>
</tr>
<tr>
<td><strong>Complement Laws</strong></td>
</tr>
<tr>
<td>$P \lor \overline{P} \iff T$</td>
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<tr>
<td>$P \land \overline{P} \iff F$</td>
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<tr>
<td><strong>Involution Law</strong></td>
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<tr>
<td>$\overline{P} \iff P$</td>
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<tr>
<td><strong>DeMorgan’s Laws</strong></td>
</tr>
<tr>
<td>$(P \lor Q) \iff P \land \overline{Q}$</td>
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<tr>
<td>$(P \land Q) \iff P \lor \overline{Q}$</td>
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</tbody>
</table>

(a) $P \land (P \lor Q) \iff P$

$$P \land (P \lor Q) \iff P$$

\(^{15}\)F is a table containing only false values, T is table containing only true values
4.3 Negation

The second part \((P \lor Q)\) contains True on each place where \(P\) is true, when we and this with \(P\) the result is \(P\).

(b) \((P \land Q) \lor \overline{P} \iff \overline{P} \lor Q\)

\[
\begin{align*}
(P \land Q) \lor \overline{P} & \iff (\overline{P} \lor Q) \land (P \lor \overline{P}) \\
& \iff (\overline{P} \lor Q) \land (T) \\
& \iff (\overline{P} \lor Q)
\end{align*}
\]

Using the distributive law, idempotent law and the identity law (respectively).

(c) \(P \land (\overline{P} \lor Q) \iff P \land Q\)

\[
\begin{align*}
P \land (\overline{P} \lor Q) & \iff (P \land \overline{P}) \lor (P \land Q) \\
& \iff F \lor (P \land Q) \\
& \iff P \land Q
\end{align*}
\]

Using the distributive law, idempotent law and the identity law (respectively).

4.3 Negation

(4.33) Simplify:

(a) \(\overline{P \land Q}\)

\[
\overline{P \land Q} = \overline{P} \lor Q
\]

(b) \(\overline{P \lor Q}\)

\[
\overline{P \lor Q} = P \land \overline{Q}
\]

(c) \(\overline{P \land \overline{Q}}\)

\[
\overline{P \land \overline{Q}} = P \lor Q
\]

These results were obtained by using DeMorgan’s Laws. We negate the entire expression, replace and/or, or/and en negate the subexpressions.

(4.34) Write the negation of each of the following statements as simply as possible

elie@de-brauwer.be 39
4.4 Conditionals, biconditionals

(a) He is tall but handsome.
   He is tall nor handsome.
(b) He has blond hair or blue eyes. He does not have blond hair and he does not have blue eyes.
(c) He is neither rich nor happy.
   He is rich or happy.
(d) He lost his job or he did not go to work today.
   He hasn’t lost his job and he went to work today.
(e) Neither Marc nor Erik is unhappy.
   Marc or Eric is happy.
(f) Audrey speaks Spanish or French but not German.
   Audrey speaks German but not Spanish nor French.

4.4 Conditionals, biconditionals

(4.35) Find the truth table of each proposition:

(a) \((\overline{P} \lor Q) \rightarrow P\)

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>\overline{P}</th>
<th>\overline{P} \lor Q</th>
<th>\rightarrow P</th>
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(b) \(Q \leftrightarrow (\overline{Q} \land P)\)

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<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>\overline{Q}</th>
<th>\overline{Q} \land P</th>
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<td>T</td>
<td></td>
</tr>
</tbody>
</table>

(4.36) Find the truth table of each proposition:

(a) \((P \leftrightarrow \overline{Q}) \rightarrow (P \land Q)\)

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>\leftrightarrow Q</th>
<th>\rightarrow P</th>
<th>\land Q</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
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<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>

| 1 | 3 | 5 | 2 | 6 | 1 |

| 3 | 4 | 5 | 2 | 6 | 1 |

elie@de-brauwer.be 40
4.5 Arguments, logical implication

(b) \((\overline{Q} \lor P) \leftrightarrow (Q \rightarrow \overline{P})\)

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>\overline{Q}</th>
<th>\lor</th>
<th>P</th>
<th>\leftrightarrow</th>
<th>Q</th>
<th>\rightarrow</th>
<th>\overline{P}</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
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<td>F</td>
<td></td>
<td>T</td>
</tr>
</tbody>
</table>

(4.37) Prove:
(a) \((P \land Q) \rightarrow R \leftrightarrow (P \rightarrow R) \lor (Q \rightarrow R)\)
(b) \(P \rightarrow (Q \rightarrow R) \leftrightarrow (P \land \overline{R}) \rightarrow \overline{Q}\)

These two statements can easily be proven by constructing the truth tables. I will not construct these truth tables here because they are rather large and mistakes are easily made.

(4.38) Determine the contrapositive of each statement:
(a) If he has courage he will win.
   If he doesn’t win, he has no courage.
   (b) Only if he does not tire will he win.
       If he tires, he will not win.

(4.39) Find:
(a) Contrapositive\(^{16}\) of \(P \rightarrow \overline{Q}\)
    \(Q \rightarrow \overline{P}\)
(b) Contrapositive of \(\overline{P} \rightarrow Q\)
    \(Q \rightarrow P\)
(c) Contrapositive of the converse\(^{17}\) of \(P \rightarrow \overline{Q}\)
    \(\overline{P} \rightarrow Q\)
(d) Converse of the contrapositive of \(\overline{P} \rightarrow Q\)
    \(P \rightarrow \overline{Q}\)

4.5 Arguments, logical implication

(4.40) Test the validity of each argument:\(^{18}\)
(a) \(\overline{P} \rightarrow Q, P \vdash \overline{Q}\)
   If \(P\) is true, then \(\overline{P} \rightarrow Q\) is true if \(Q\) is false, in this case the argument is valid, but \(\overline{P} \rightarrow Q\) is also true if \(Q\) is true, in this case the conclusion would be false!

---

\(^{16}\)If \(A \rightarrow B\) then \(\overline{B} \rightarrow \overline{A}\) is the contrapositive

\(^{17}\)If \(A \rightarrow B\) then \(B \rightarrow A\) is the converse

\(^{18}\)An argument \(P_1, P_2, \ldots, P_n \vdash Q\) is valid if \(Q\) is true whenever all the premises \(P_1, P_2, \ldots, P_n\) are true, an argument that is not valid is called a fallacy.
4.5 Arguments, logical implication

(b) \( \overline{P} \rightarrow Q, Q \vdash P \)

First Q has to be true. For \( \overline{P} \rightarrow Q \) to be true, P can be either true of false. If P is false, the conclusion would be false!

This can all be proven using truth tables, but I use a more intuitive method.

(4.41) Test the validity of each argument:

(a) \( P \rightarrow Q, R \rightarrow \overline{Q} \vdash R \rightarrow \overline{P} \)

Always valid

(b) \( P \rightarrow \overline{Q}, \overline{R} \rightarrow \overline{Q} \vdash P \rightarrow \overline{R} \)

If P is false R can be true or false (starting from the conclusion), since P is false \( P \rightarrow \overline{Q} \) is always true, so Q can be either true or false. If R is false and Q is true then \( \overline{R} \rightarrow \overline{Q} \) is false, this gives a fallacy!

(4.42) Translate into symbolic form and test the validity of the argument:

(a) If 6 is even, then 2 does not divide 7

Either 5 is not prime or 2 divides 7

But 5 is prime

... Therefore, 6 is odd(not even).

\( P \rightarrow \overline{Q}, \overline{R} \lor Q, R \vdash \overline{P} \), valid

(b) Roses are red

Roses are blue

... Therefore, roses are red if and only if they are blue.

\( Q, P \vdash P \rightarrow Q \), valid

(c) If I work, I cannot study

Either I work, or I pass mathematics

I pass mathematics

... Therefore, I studied

\( P \rightarrow \overline{Q}, P \lor R, R \vdash Q \), fallacy

(d) If I work, I cannot study

Either I study or I pass mathematics

I worked

... Therefore, I passed mathematics

\( P \rightarrow \overline{Q}, P \lor R, P \vdash R \), valid
4.5 Arguments, logical implication

(4.43) Show that:\textsuperscript{19}

(a) $P \land Q$ logically implies $P$
This is valid since $P \land Q$ can only be true if $P$ is also true.

(b) $P \lor Q$ does not logically imply $P$
This is valid since $P \lor Q$ can also be true is $P$ is false and $Q$ is true.

(4.44) Show that:

(a) $Q$ logically implies $P \rightarrow Q$
Valid since true $\rightarrow$ true is true and false $\rightarrow$ true is true.

(b) $\overline{P}$ logically implies $P \rightarrow Q$.
Valid since false $\rightarrow$ true is true and false $\rightarrow$ false is true.

(4.45) Determine those propositions which logically imply:

(a) A tautology
Every proposition logically implies a tautology since a tautology is always true whenever the proposition is true.

(b) A contradiction
Only a proposition that is never true can logically imply a contradiction, thus only a proposition that is a contradiction logically implies a contradiction.

\textsuperscript{19}A proposition $P(p,q,\ldots)$ is said to logically imply a proposition $Q(p,q,\ldots)$ if $Q(p,q,\ldots)$ is true whenever $P(p,q,\ldots)$ is true
5 Algorithms, Flowcharts, Pseudocode programs

5.1 Computer programs, variables, constants

(5.24) BASIC, COBOL, FORTRAN are:

(a) Compiler languages True, these languages are not written in low level language and have to be compiled or interpreted in order to be understood by the computer

(b) High-level languages True, see a

(c) Machine-independent languages True, only machine language is machine dependent

(d) All of the above True

(5.25) A compiler translates a source program into an object program True, and the linker combines the object program(s) to an executable

(5.26) A data item whose value may change during the course of a program is called a variable, whereas a data item whose value normally does not change during the course of a program is called a constant.

(5.27) Which of the following are unacceptable as variable-names in BASIC

(a) B7 acceptable
(b) C34 unacceptable, more than two characters
(c) 3D unacceptable, starts with a digit
(d) F% unacceptable, special character
(e) BONUS unacceptable, more than two characters
(f) EMPLOYEE unacceptable, more than two characters
(g) Z acceptable
(h) A*B unacceptable, more than two characters
(i) NUMBER unacceptable, more than two characters
(j) Y2 acceptable

(5.28) Which of the character strings in Problem 5.27 are unacceptable as variable-names in Fortran?

c is unacceptable since it starts with a digit, d is unacceptable since it contains a special character, f is unacceptable since its name is too long and h is also unacceptable since it contains a special character.

(5.29) Which of the following are unacceptable as numeric constants?

20 Variable names in BASIC may not exceed two characters in length
5.1 Computer programs, variables, constants

(a) \(333.444\) acceptable
(b) \(4,000.00\) unacceptable
(c) \(7,9999\) acceptable
(d) \(\$95.75\) unacceptable, invalid character
(e) \(234.5E-15\) acceptable
(f) \(2\,000\) unacceptable, invalid character
(g) \(50\,£\) unacceptable, invalid character
(h) \(7.7E111\) unacceptable too large exponent

(5.30) Write a BASIC\(^{21}\) formula for each algebraic expression:

(a) \(x^2 + y^2\)

```basic
double result, x, y;
// read x and y
result = x*x + y*y;
```

(b) \((x + y)^2\)

```basic
double result, x, y;
// read x and y
result = (x+y)*(x+y);
```

(c) \(\frac{2ab}{c+d}\)

```basic
double result, a, b, c, d;
// read a,b,c,d
result = 2*a*b/(c+d);
```

(d) \(t^{n+1}\)

```basic
double result, t, n;
// read t and n
result=1;
for(int i=0;i<=n;i++){
    result*=t;
}
```

(e) \(\frac{x}{y^2}\)

\(^{21}\)I will write it in C/C++, since I’m not such a big BASIC fan
5.2 Flowcharts, pseudocode programs

```
double result, x, y, z;
// read x, y and z
result = x/(y*z);
```

(f) \((x^2 + 2xy - y^2)^5\)

```
double result, x, y;
// read x and y
result = x*x + 2*x*y - y*y;
for(int i=1;i<5;i++){
    result*=result;
}
```

(5.31) Write a FORTRAN formula for \(t^{n+1}\):

(a) Using parentheses : \(t^{**}(n+1)\)
(b) Without parentheses : \(t*t**n\)

(5.32) Evaluate each BASIC expression:

(a) \(5+3*8-4/2 = 27\)
(b) \((5+3)*(8-4)/2 = 16\)
(c) \(5+3*((8-4)/2) = 11\)
(d) \((5+3)*((8-4)/2) = 16\)
(e) \((5+3)*(8-4/2) = 48\)
(f) \(5+(3*8-4)/2 = 15\)

(5.33) Evaluate each BASIC expression:

(a) \(5+2\uparrow -2*3\uparrow 2 = -5\)
(b) \(5+2\uparrow (3-2)*3\uparrow 2 = 23\)
(c) \((5+2)\uparrow (3-(2*3))\uparrow 2 = 307\)
(d) \((5+(2\uparrow 3-2)*3)\uparrow 2 = 59\)

5.2 Flowcharts, pseudocode programs

(5.34) The perimeter \(P\) and the AREA of a triangle whose sides have lengths \(a, b, c\) are given \(P = a + b + c\) and \(AREA = \sqrt{s(s-a)(s-b)(s-c)}\) where \(s \equiv \frac{a+b+c}{2}\). Write a pseudocode program with input \(a, b, c\) and output \(P\) and \(AREA\)
5.2 Flowcharts, pseudocode programs

read A,B,C
P = A + B + C
S = P/2
AREA = sqrt(S(S-A)(S-B)(S-C))
write P, AREA
end

(5.35) An automobile dealership uses CODE=1 for new automobiles, uses CODE=2 for used automobile and CODE = 3 for separate accessories. A salesman’s commissions are as follows: on a new automobile, 3% of the selling price but with a maximum of $300; on a used automobile, 5% of the selling price but with a minimum of $75; on accessories 6%. Write a pseudocode program with input CODE and PRICE and output COMMISSION

read CODE, PRICE
IF CODE =1
    COMMISSION = 0.03 * PRICE
    IF COMMISSION > 300
        COMMISSION = 300
    ENDIF
ELSE IF CODE = 2
    COMMISSION = 0.05 * PRICE
    IF COMMISSION < 75
        COMMISSION = 75
    ENDIF
ELSE
    COMMISSION = 0.06 * PRICE
ENDIF
write COMMISSION
END

(5.36) The monthly charge for local telephone calls is as follows:

- $8.00 for up to 100 calls
- plus 6 cent per call for any of the next 100 calls
- plus 4 cent per call for any calls beyond 200

Write a pseudocode program with input the number of LOCAL calls and output CHARGE.

read LOCAL 

elie@de-brauwer.be
5.2 Flowcharts, pseudocode programs

IF LOCAL <= 100
  CHARGE = 8.00
ELSIF LOCAL <= 200
  CHARGE = 8.00 + 0.06 * (LOCAL-100)
ELSE
  CHARGE = 8.00 + 0.06 * 100 + 0.04 * (LOCAL - 200)
ENDIF
write CHARGE
END

(5.37) Three positive numbers a,b,c can be the lengths of the sides of a triangle provided each number is less than the sum of the other two (i.e. \( a < b + c, b < a + c, c < a + b \)). Write a pseudocode program with input a,b, c and output 'YES' or 'NO', according as a triangle can be formed.

read A,B,C
IF (a<b+c AND b<a+c AND c < a+b)
  write 'yes'
ELSE
  write 'no'
ENDIF
END

(5.38) Find the output of the flowchart in Fig 5-25 (book page 119) for the inputs:

(a) X=15, Y=20
(b) X=15, Y=5
(c) X=2, Y=18

The flowchart contains an algorithm that would like this in C++:

```
#include <iostream>
using namespace std;

int main(void){
  int a,b;
  \read a and b
  if(a<b){
    a = a + 2*b;
    if(a<20){
```
5.3 Loop structures

Sometimes C++ will be used instead of pseudocode in the most simple cases. When no file IO is needed C++ is as easy to understand as pseudocode (in my honest opinion).

5.39 Write the pseudocode program corresponding to the flowchart stated in question 5.38.

See question 5.38.

5.40 In which process below can the two statements be interchanged without changing the resulting values of the variables?

(a) C=A, D=B, possible
(b) C=A, C=B, C gets overwritten
(c) C=A, D=A, possible
(d) C=A, A=B, A gets overwritten
(e) C=A, D=C, changed variable C is used
(f) C=A, A=C, changed variable C is used

5.41 Problems 5.41 and 5.42 are skipped because nothing worldshaking happens.

5.3 Loop structures

Sometimes C++ will be used instead of pseudocode in the most simple cases. When no file IO is needed C++ is as easy to understand as pseudocode (in my honest opinion).

5.43 Write a pseudocode program which lists the names of the A students (those with scores of 90 or above) and finds the number and percentage of such students.

```
totalstudents = 0
astudents = 0
read NAME, SCORE
DOUNTIL EOF
    IF SCORE >= 90
        write a and b
        astudents = astudents + 1
        totalstudents = totalstudents + 1
write astudents and totalstudents
```

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### 5.3 Loop structures

```plaintext
write NAME
astudents = astudents + 1
ENDIF
totalstudents = totalstudents + 1
ENDDO
write 'Number of a students is ' astudents
write 'This is ' astudents/totalstudents * 100 ' percent'
END
```

(5.44) Write a pseudocode program using the DOWHILE structure that finds all pairs of positive integers \(m, n\) such that \(m^2 + 2n^2 < 100\).

```plaintext
#include <iostream>
using namespace std;

int main(void){
    for(int i=0; i<=9; i++){
        for(int j=0; i*i+2*j*j<100; j++){
            cout << i << "," << j << endl;
        }
    }
    return 0;
}
```

(5.45) Write a pseudocode program which finds the largest and smallest of 25 distinct numbers

```plaintext
#include <iostream>
using namespace std;

int main(void){
    int input, max, min;
    cout << "Please enter a number" << endl;
    cin >> input;
    max = input;
    min = input;
    for(int i=0; i<24; i++){
        cout << "Please enter a number" << endl;
        cin >> input;
        if(input > max){
            max = input;
        } else if(input < min){
            min = input;
        }
    }
    return 0;
}
```
5.3 Loop structures

```cpp
min = input;
}
}
cout << max << "," << min << endl;
return 0;
}

(5.46) Write a pseudocode program which finds the second-largest of 25 distinct numbers

```cpp
#include <iostream>
using namespace std;

int main(void){
    int input, max, secondmax;
cout << "Please enter a number" << endl;
cin >> input;
    max = input;
cout << "Please enter a number" << endl;
cin >> input;
    if(input > max){
        secondmax = max;
        max = input;
    }else{
        secondmax = input;
    }
    for(int i=0; i<23;i++){
        cout << "Please enter a number" << endl;
cin >> input;
        if(input > max){
            secondmax = max;
            max = input;
        }else if(input > secondmax){
            secondmax = input;
        }
    }
cout << secondmax << endl;
return 0;
}
```

(5.47) Suppose that y is the following function of t: $y = -2t^3 - t^2 - 37t + 36$. Write a pseudocode program which outputs y correspond to each t from -5
5.3 Loop structures

to 5 in steps of 0.25, and finds the maximum and the minimum of the calculated values of y.

```cpp
#include <iostream>
using namespace std;

int main(void)
{
    int max, min, res;
    max = 2*(-5*-5*-5) - (-5*-5)-37*-5 + 36;
    min = max;
    for(int i = -4.8; i<=5; i=i+0.2){
        res = 2*t*t*t - t*t - 37*t + 36;
        if(res>max){
            max = res;
        }else if(res < min){
            min = res;
        }
    }
    cout << min << "," << max << endl
    return 0;
}
```

(5.48) Recall that n! is defined for a positive integer n by $n! = 1 \cdot 2 \cdots \cdot n$. Write a pseudocode program that calculates n!.

```cpp
#include <iostream>
using namespace std;

int main(void)
{
    int n, res;
    cout << "Please enter n " << endl;
    cin >> n;
    res = 1;
    for(int i=2; i<n; i++){
        res *= i;
    }
    cout << res << endl;
    return 0;
}
```

(5.49) Find the number of cycle and the value of the index K for each cycle if a DO loop is headed by:

elie@de-brauwer.be
5.3 Loop structures

(a) Do $K=1$ to $20$ by $3$ in $7$ times
(b) Do $K=4$ to $9$ by $2$ in $3$ times
(c) Do $K=7$ to $5$ by $2$ not
(d) Do $K=15$ to $10$ by $-2$ in $3$ times
6 Sets and Relations

6.1 Sets, set operations

(6.32) Which of the following sets are equal?22?

(a) \{1, 2\}
(b) \{1, 3\}
(c) \{2, 1\}
(d) \{3, 1, 3\}
(e) \{1, 2, 1\}
(f) Soon

6.2 Algebra of sets, venn diagrams

6.3 Classes of sets, partitions

6.4 Product sets

6.5 Relations

6.6 Equivalence relations

6.7 Functions

---

22 Two sets are equal when they contain the same elements
7 Boolean Algebra, Logic gates

7.1 Boolean algebra

7.2 Logic gates
8 Simplification of Logic Circuits

8.1 Minimal sums, Karnaugh maps
8.2 Minimal and-or circuits
9 Vectors, Matrices, Subscripted Variables

9.1 Vectors
9.2 Matrix operations
9.3 Square matrices
9.4 Subscripted variables
10 Linear Equations

10.1 Linear equations in one and two unknowns
10.2 Square systems of linear equations
10.3 Matrices and linear equations
10.4 Determinants and linear equations
11 Combinatorial Analysis

11.1 Factorials, binomial coefficients
11.2 Permutations, partitions
11.3 Combinations
11.4 Tree diagrams
12 Probability

12.1 Probability spaces
12.2 Conditional probability, independence
12.3 Independent trials
13 Statistics, Random Variables

13.1 Statistics
13.2 Random variables, expectation
14 Graphs, Directed Graphs, Machines

14.1 Graphs, connectivity
14.2 Tree graphs
14.3 Directed graphs
14.4 Finite state machines